

Department of ECE Breadth Exam
Question 1: Engineering Mathematics
Problem 3: Probability Theory & Random Variables

Please write neatly and show all your work.

1. *Concepts.*

- (a) Let X and Y be two random variables such that $Y = -3X + 7$. Then, ρ_{XY} , the correlation coefficient between X and Y , equals 1. True or False? *Justify.*
- (b) Prove that the variance of the sum of two random variables equals the sum of their variances if and only if the random variables are uncorrelated.

2. Let the random variables X and Y have the following joint PDF:

$$f_{XY}(x, y) = \begin{cases} 4xy, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}.$$

- (a) Find the mean and variance of X .
- (b) Find the mean and variance of Y .
- (c) Find the variance of $X + Y$.