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Electrical and Computer Engineering
Fall 2023 BREADTH EXAM

Problem 1 | Engineering Mathematics | P2: The Z-transform

X(z) denotes z-transform of x[n] $X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$

X(ω) denotes the DTFT of x[n] $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

u[n] is the unit step, δ[n] is the unit sample ω denotes frequency in rad/sample

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***Circle* the Best Answer** **Show All Work Even For Multiple Choice**

1. The right-sided sequence h(n) with z-transform $H(z) = \frac{5z-1}{z^2+4z+4}$ is BIBO stable.

   a) True b) False

2. If a filter has $H(z) = \frac{z^2+2z-2}{4z^2+1}$; $|z|>5^{-1/2}$, then the dc response of the filter at ω=0 is

   a) 0 b) 1/5 c) 1/4 d) 1/3 e) none above

3. The ROC $0.2 < |z| < 0.5$ could be associated with a BIBO stable 2-sided sequence.

   a) True b) False

4. If $H(z) = \frac{z^2-z/3-1/2}{4z^2-1}$, then the poles of H(z) are at z=

   a) j/4, -j/4 b) 1/2, -1/2 c) j/2, -j/2 d) none above

5. One of the zeroes of the z-transform $H(z) = 1 + z^4$ is at z =

   a) $e^{j\pi/2}$ b) $e^{j5\pi/8}$ c) $e^{-j5\pi/8}$ d) $e^{j5\pi/4}$ e) none above

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6. The z-transform of h[n]= δ[n-1] - 4δ [n-3] is H(z) =

a) $1 + 4z^{-3}$; |z|>0.25 b) $\frac{z^2-4}{z^3}$; |z|>0

c) $\frac{z^{-1}-4z^{-3}}{z^{-1}}$; |z|>0.125 d) none above

7. The z-transform of h[n]= $(1/2)^n$ u[n] is H(z) =

a) $\frac{z}{z-1/2}$;|z|>1/2 b) $\frac{2}{z-1/2}$;|z|>1/2 c) $\frac{1}{2z-1}$;|z|>1/2 d) none above

8. The system with z-transform $H(z) = \frac{50(z+1)(z-1)}{100z^2+1}$; |z|>0.1 would be best described as

a) lowpass b) highpass c) bandpass

9. If a filter has $H(z) = \frac{z^2-2z-1.9}{z^3+1/8}$ and ROC |z|>1/2, then the first 3 points of h[n]=

a) {1, -2, 1.9} b) {1, -2.0, 0} c) {0, 1, -2.0} d) none above

10. The z-transform of u[n-1] - u[n-3] is

a) $1 - 2z^{-1} - 3z^{-2}$; |z|>1/5 b) $z^{-1} + z^{-2}$; |z|>0

c) $1 - z^{-1} - z^{-2}$; |z|>0 d) none above

11. If a system has $H(z) = \frac{4z-1/5}{4z^2-1}$ and ROC |z|>1/2 then, then h[1] =

a) 0 b) 4/5 c) 1 d) 5/4 e) none above

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12. If a filter has impulse response h[n]= u[n-1] - u[n-5], the filter response at frequency $\omega=\pi$ is $H(\omega)|_{\omega=\pi}$ =

a) -1 b) e^{-j4} c) 1 d) j e) none above

13. If a causal filter has $H(z) = \frac{3z^2-z+2}{5z^2-z+3}$ then the response of the filter at $\omega=\pi$ is

a) 0 b) 4/7 c) 3/5 d) 2/3 e) none above

14. If $H(z) = \frac{5}{z+1/2} + \frac{1}{z-1/2}$; |z|>1/2, then the zero of H(z) is at z=

a) 1/3 b) 1/4 c) 1/2 d) 1 e) none above

15. If $H(z) = \frac{4z}{2z+1}$; |z|>1/2, then h[n]=

a) $(1/8)^n$ u[n] b) $4(-1/2)^n$ u[n]

c) $2(-1/2)^n$ u[n] d) $2(1/2)^n$ u[n] e) none above

16. If a filter has impulse response h[n]=$(-1/5)^{n-2}$ u[n-2], the dc response of the filter is $H(\omega)|_{\omega=0}$ =

a) 2/5 b) 5/6 c) 4/5 d) $0.2e^{-j2\omega}$ e) none above

17. A filter with $H(z) = \frac{z^2+4z+1}{z^2+4}$; with ROC |z|<2 is

a) unstable b) left-sided c) two-sided d) causal e) none above

18. If $Y(z)=1+z^{-1}$; |z|>0, and $X(z)= 1+z^{-2}$; |z|>0 , then the convolution $x[n] * y[n]$=

a) $\delta[n] +\delta[n-1] +\delta[n-2] +\delta[n-3]$ b) $\delta[n] +2\delta[n-1] +\delta[n-2]$ c) $\delta[n-1] +\delta[n-2]$ d) none above

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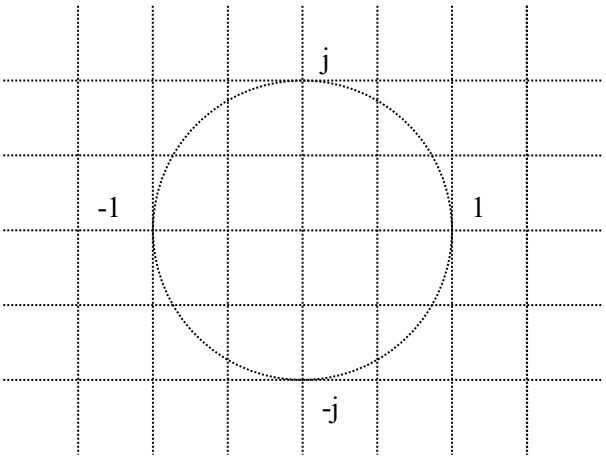

For the following questions:

$$H(z) = \frac{z^2 + 1/16}{z^2 + z/4 - 1/8}$$

19. Sketch the poles and zeroes in the figure above.

20. Assuming a causal system, sketch the region of convergence in the figure above.

Show your work for the 2 above problems in the space below.