Control Number: ____________

Electrical and Computer Engineering
Fall 2023 BREADTH EXAM

Problem 1 | Engineering Mathematics | P1: Fourier-transform

Show all your work (derivations and calculations). Clearly indicate the question and part number for all your answers. Label all your plots

Formulas:

Integration by parts: $\int udv = uv - \int vdu$.

Exponential Fourier Series: $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$.

Fourier series coefficients: $a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$.

Differentiation property: When a_k are the complex exponential Fourier series coefficients for $x(t)$, $jk\omega_0 a_k$ are the complex exponential Fourier coefficients for $\frac{dx(t)}{dt}$.

Fourier transform: $X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$.

Inverse Fourier transform: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$.

Parseval Equality: $\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$.

Multiplication property: $s(t)p(t) \leftrightarrow \frac{1}{2\pi}S(j\omega) * P(j\omega)$, where $*$ denotes the convolution operation.

1. A periodic signal $x(t)$ of period $T = 4$ is defined over a period by

$$x(t) = \begin{cases} 0 & -2 < t < -1 \\ 1 & -1 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

   (a) What is the value of fundamental frequency ω_0 in rad/s?

   (b) Compute the complex exponential Fourier series coefficients for $x_1(t) = \frac{dx(t)}{dt}$

   (c) Compute the complex exponential Fourier series coefficients for $x(t)$ using your results from (b)

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2. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 4$. The nonzero complex exponential Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1} = 2 \ , \ a_3 = a_{-3}^* = 4j.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

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3. You are told that the spectrum of the signal $g(t) = \frac{\sin(At)}{\pi t}$ is given by

$$G(j\omega) = \begin{cases} 1, & -A \leq \omega \leq A \\ 0, & \text{elsewhere.} \end{cases}$$

Now let

$$h_1(t) = \left(\frac{\sin(\frac{\pi t}{2})}{\pi t}\right)\left(\frac{\sin(\pi t)}{\pi t}\right).$$

(a) Compute the total energy E_∞ of $g(t)$, where $E_\infty = \int_{-\infty}^{+\infty} |g(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |G(j\omega)|^2 d\omega$.

(b) Determine the frequency response $H_1(j\omega)$ for $h_1(t)$; using the Fourier transform multiplication property. (The formula is given to you on the first page).

(c) Sketch the magnitude $|H_1(j\omega)|$.

(d) Let $h_2(t) = h_1(t)\cos(\pi t)$, determine the frequency response $H_2(j\omega)$ for $h_2(t)$. (Use the multiplication property given)

(e) Sketch the magnitude $|H_2(j\omega)|$.