

**Electrical and Computer Engineering
Fall 2023 BREADTH EXAM**

Question 1Engineering MathematicsP5: Counting, Recurrence
Relations, and Graphs

1. (40%) An embedded system has 4096 bytes of main memory that is organized into 128 cache lines (bytes addressed from address 0 to 31 form the first cache line, 32 to 63 form the second cache line, etc.)

A direct-mapped cache memory for this system holds 8 cache lines.

- (a) Is the mapping from main memory to cache memory *one-to-one* (i.e. an injection)? *Justify your answer.*
- (b) Is the mapping from main memory to cache memory *onto* (i.e. a surjection)? *Justify your answer.*
- (c) Assuming the processor accesses address 0, causing the the first cache line to be transferred into cache memory (no other cache lines have been loaded). The next memory address the processor accesses is randomly selected from main memory. What is the probability of a cache hit? (By ‘hit’ we mean that the data will be found in cache memory.) *Justify your answer.*
- (d) If memory is always accessed sequentially (address 0, 1, ...), then describe a function that will map main memory addresses to cache locations such that the probability of a cache miss is minimized. (By ‘miss’ we mean that the processor accesses an address in memory and data is not already present in the cache.) *Justify your answer.*

HINT: Use bit fields in the address to describe the domain and range of your function; then describe when a miss is possible.

2. (20%) Consider a Boolean system of three variables (x , y , and z).

- (a) How many distinct values can x have?
- (b) For a Boolean function $f(x, y, z)$, how many distinct values can f have?
- (c) For all possible Boolean functions f , how many different combinations of the three variable are there?
- (d) For the Boolean function $f(x, y, z) = x + yz$, enumerate all possible combinations of inputs and the associated output of f in a table.
- (e) When the output $f(x, y, z) = 1$, that combination of inputs is called a minterm. When the output $f(x, y, z) = 0$, that combination of inputs is called a maxterm. What is the relationship between the number of minterms ($|\{m_i\}|$), maxterms, ($|\{M_i\}|$) and total number of all combinations of inputs (*rows*)?

3. (40%) Suppose an algorithm’s run time is related to the size of its input as follows.

$$R(n) = \begin{cases} 1^2 & n = 1 \\ R(n-1) + n^2 & n > 1 \end{cases}$$

- (a) Is this a recurrence relation?
- (b) Is $R'(n)$ (below) the closed form of $R(n)$?

$$R'(n) = \frac{n(n+1)(2n+1)}{6}$$

If it is the closed form, prove it.

If it is *not* the closed form, prove it.

Show your work!

- (c) What is the growth rate of $R(n)$ in terms of Big-Oh notation?