Control Number _____

Electrical and Computer Engineering
Fall 2023 BREADTH EXAM

Problem 1 Engineering Mathematic P1: Differential Equations

Problem 1 (25 points total) – (20 pts) Solve this ordinary differential equation, $y'+ y = 5cos(t)$, *for y(t), with y(0) = 1; Discuss qualitatively the behavior at large t (5pts). Does the large t behavior depend on the initial condition y(0)?*

Problem 2 (35 points) – Solve equation $y'' + \lambda y = 0$ for y(x), with y(0) = y(1) = 0. Determine the allowed values of the parameter λ for non-trivial solutions (i.e., non-zero solution) of y(x). Normalize the solution: $\int_0^1 y^2\, dx = 1$. Note that this problem seeks for standing wave solutions with two end points at x = 0 and x = 1.

Problem 3 (40 points total) – If the undamped harmonic oscillator is applied an extra oscillating force so that the equation to solve is $md^2x/dt^2 = - kx + F_{ext}(t)$, where the external force is $F_{ext}(t) = F_0 \cos(\omega_0 t)$, F_0 is a constant, and $k/m = \omega^2$. Assume that $\omega_0 \neq \omega = \sqrt{(k/m)}$. (25 pts) Find the solution x(t) with the initial conditions x(0) = 0. Discuss three special cases: 1) (5 pts) $\omega_0 = 0$; 2) (5pts) $\omega_0 >> \omega$; 3) (5pts) t is small with an arbitrary ω_0 (i.e., showing the leading t-dependent term in the general solution). In each case explain the underlying physics.