UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE
Department of Electrical and Computer Engineering

EXPERIMENT 6 – UNRAVELING CONVOLUTION

OBJECTIVES

After completing this experiment, the student will have

- Carry out a step-by-step dissection of the convolution process in a discrete-time system and discover the convolution formula.
- Visualized the convolution as a running average of successive values of the input signal.
- Observe a special property applying to sinewaves.
- Demonstrate the operation of a filter in the time domain.

MATERIALS/EQUIPMENT NEEDED

NI ELVIS II
EMONA SIGEx Signal & Systems add-on board
Assorted patch leads
Two BNC – 2mm leads

INTRODUCTION

For many students, the first encounter with convolution is an abstract mathematical formula in a textbook. This experiment offers a more ilustrative experience. By tracing the passage of some basic signals through a simple linear system, you will be able to observe the underlying process in action, and, with a little math, discover a formula as it emerges from the hardware.

In this experiment students will patch up a delay line with two unit delays and three taps with independently adjustable gains as in Figure 6-1. In the first exercise students will set these gains to given values, and observe the output when the input is a single pulse (more exactly, a periodic sequence of single pulses). This is an important preliminary as it introduces the unit pulse response.

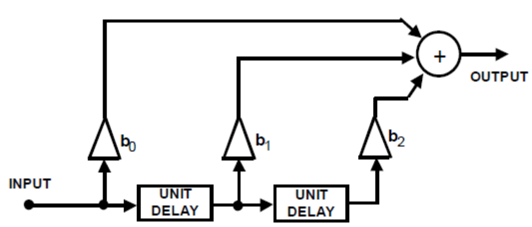


Figure 6-1 General Block Diagram of the System

Then, the students will observe the output when a pair of adjacent pulses is used as input. This near trivial example provides with a springboard to the general case. It will demonstrate how convolution operates as an overlapping and superposition of unit pulse responses. A second more general input sequence is then used to reveal a deeper insight and to provide a vehicle for setting up the key formulas. In the remainder of the experiment sinewaves will be used as inputs, this will lead students to the rediscovery of the special role of sinusoids in linear time-invariant systems. The final exercise to be carried out will unravel a disappearing act. It should about 40 minutes to complete this experiment.

PRELAB

1. Refresh your basic trigonometry: you will need $\sin(\omega t) + Acos(\omega t + \emptyset)$ expressed as a single sinusoid.
2. If any of the modules is unfamiliar, spend a little time with the SIGEx User Manual. This will give you a head start in setting up the lab.

PROCEDURE

Setting up the NI ELVIS/SIGEx Board

1. Turn on the PC (if not on already) and wait for it to fully boot up.
2. Turn on the NI ELVIS unit but not the Prototyping Board switch yet. You should observe the USB light turn on (top right corner of ELVIS unit).
3. Turn on the NI ELVIS Prototyping Board switch to power the SIGEx board. Check that all three power LEDs are on. If not call the instructor for assistance.
4. Launch the "SIGEx Rx_x.exe" Main VI.
5. When you're asked to select a device number, enter the number that corresponds with the NI ELVIS that you're using.
6. You're now ready to work with the NI ELVIS/SIGEx bundle.
7. Select the Exp 5 tab on the SIGEx SFP.
   Note: To stop the SIGEx VI when you've finished the experiment, it's preferable to use the STOP button on the SIGEx SFP itself rather than the LabVIEW window STOP button at the top of the window. This will allow the program to conduct an orderly shutdown and close the various DAQmx channels it has opened.

System Setup and Unit Pulse Response

1. Connect the model in Figure 6-2. The settings are as follows:
   a. PULSE GENERATOR: FREQUENCY=1000Hz; DUTY Cycle:0.5(50%)
   b. SEQUENCE GENERATOR: Dips UP/UP
   c. SCOPE: Timebase: 10ms; Rising edge trigger on CH0; Trigger level=1V
2. The required signal appears at the SEQUENCE GENERATOR SYNC output as a 5V signal and needs to be reduced in amplitude using the available a_0 GAIN function . Using the scope, check that you have a periodic sequence of a single 1V pulse in a frame of 31 pulse periods. Confirm the pulse width is 1ms. Adjust the a_0 gain value to have a pulse amplitude of 1V precisely. Adjust the SCOPE trig level to suit.

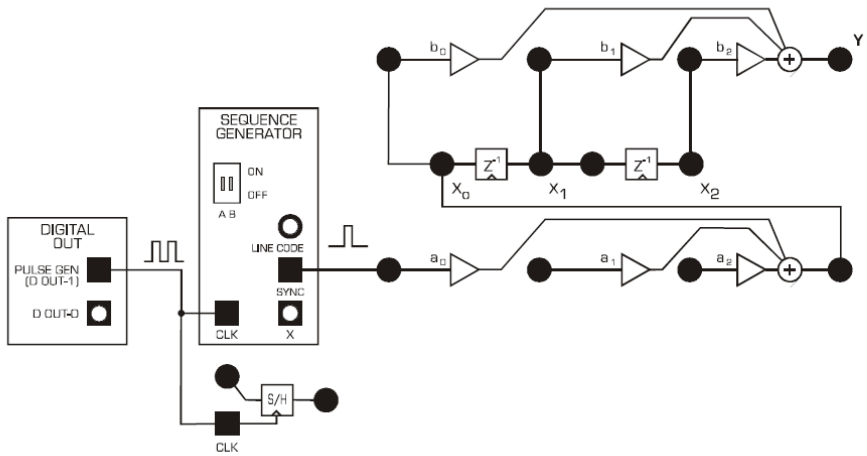


Figure 6-2 Diagram of System with the Pulse Generator

3. Note that as the incoming pulse is being clocked by the same clock as the unit delay blocks, the pulse is already aligned with the unit delay clock and is thus already a discrete signal. For this reason it can be input directly into the unit delay without use of the S/H block.
4. Before proceeding with the examination of the system response, the delay line "tap" gains must be set. For the first case we shall use b_0 = 0.3, b_1 = 0.5 and b_2 = -0.2 (see Figure 6-1). These settings have been chosen arbitrarily as interesting and varied values for this exercise. Adjust each gain in turn on the SigEx software and use the scope to confirm your settings.
5. Record (describe) the procedure you used for confirming each GAIN.
6. Display the delay line input signal (i.e. at the first z-1 block input) and the ADDER output signal. Measure and record the amplitude of each pulse in the output sequence.
7. Note that the system output is a sequence of three contiguous pulses with amplitudes in the same ratio as the adder input gains. Could this have been predicted from Figure 6-1?
8. From your measurements, show that the unit pulse h(0) = b0 , h(1) = b1 , h(2) = b2 in this example.

The Superposition Sum

1. Adjust the SEQUENCE GENERATOR DIP switches to position UP:DOWN to select the sequence of two contiguous pulses. Using the same gain settings as in the previous part, observe the output signal. Note that it consists of four nonzero pulses per frame. Measure and record the amplitude of each pulse.
2. Verify that the output sequence is simply the sum of two offset unit pulse responses. Use Graph 6-1below to show your computation.
3. What is meant by "superposition"? Discuss how this exercise relates to superposition and the "additivity" principle.
4. What do you expect to see if this exercise were expanded to two or more contiguous pulses? Explain.

Rectified Sinewave at Input

In this part the input is a little more interesting than before; a sequence of three or four pulses of different amplitude. We obtain the source of this signal from the ANALOG OUT DAC-0. We then pass this analog signal to the SAMPLE/HOLD block to be sampled and this becomes our discrete sequence of pulses. Note that the PULSE GENERATOR and DAC signal generator share the same internal clock and hence no slippage occurs in the scope displays.

Note that although the signal is sampled and becomes “discrete” it has not become a “digital” signal. This is an important distinction. Rather it now exists as sequential discrete samples of the original signal. More about sampling and its implications in several later experiments.

1. Connect the circuit shown in Figure 6-3. The settings are as follows:
   a. PULSE GENERATOR: FREQUENCY=800Hz; DUTY Cycle:0.5(50%)
   b. SCOPE: Timebase: 10ms; Rising edge trigger on CH0; Trigger level=1V
2. Confirm that the sinewave from the DAC-0 is 100Hz, 2V peak, before entering the RECTIFIER. We will treat the sinewave as a continuous signal and ignore the very small steps present as these have no consequence to our procedure.
3. Note the amplitude of the half wave rectified sine and explain why its amplitude is reduced relative to the input?

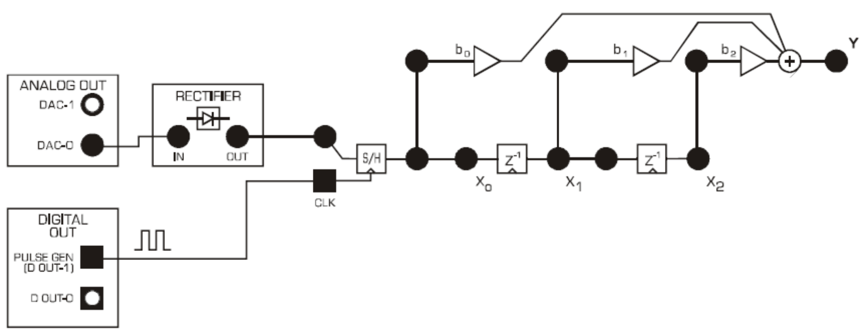
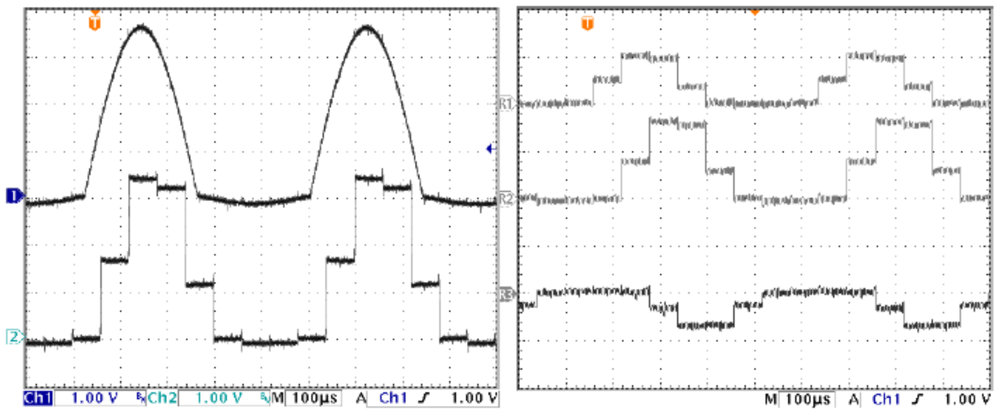


Figure 6-3 SigEx Diagram for Rectified Sinewave Input

4. Maintaining the values for the b gains as $b_0 = 0.3$, $b_1 = 0.5$ and $b_2 = -0.2$, display the input (i.e., SAMPLE-HOLD output as shown in Figure 6-4a) and output signals. Sketch the original half-rectified sinusoid and discrete output from the SAMPLE/HOLD in Graph 6-2.
5. Confirm that there are 8 samples of each half wave sine input. This is expected as the sampling clock is 800Hz and the input sinewave is 100 Hz.



(a) **(b)**

Figure 6-4 Example: (a) Input Signal and 4-level Sampled Signal (b) b_0, b_1 & b_2 Inputs to Adding Junction

Note there are four nonzero pulses in the delay line input sequence and six in the summer output. The reason for this will emerge as we proceed. Again, we will carry out a deconstruction of the output signal in terms of the time offset contributions, i.e. we will trace the output pulse amplitudes in terms of the input values. We could use the same method previously used, however, there is an interesting alternative. We will separately observe and compare the three individual contributions into the output ADDER, i.e. the signals added through gains b_0, b_1 and b_2 in turn. Examples of these signals are shown in Figure 6-4b.

6. We begin with the contribution through b_2. Temporarily disconnect the leads corresponding to inputs b_0 and b_1. Observe and record the input and output signals in Graph 6-2. Confirm for yourself that this result is as expected. View the sampled input on one scope channel and the individual output on the other scope channel.
7. Now repeat for the b_1 and b_0 contributions. Only the outputs need be recorded since the same input is used. Again, verify that the results are as expected. When completed, you will have three scaled replicas of the input with time offsets.
8. For each time slot, sum the contributions of the three output records and plot the result. Verify that this agrees with the output signal produced when the three leads are reconnected to the adder inputs.
9. How does this process relate to the principle of "superposition"?
10. In summary, we have just passed a sequence of discrete values (our sampled input) through a system with a particular response (a series of unit delays with multiplying taps) to produce an accumulation (the adder) of discrete product terms which are then output. Each one of these individual contributions is the scaled and shifted version of the unit pulse response of this system. Another way of thinking about this system as it being a series of weighting coefficients.
11. Now think about representing this process mathematically. An obvious way to start is to write a separate formula for the six nonzero output pulse amplitudes. Let us name them y(1), y(2), …, y(6). Label them on your Graph 6-2. Pay attention to the orientation you use.

12. Each consists of a sum of three products, i.e., of a tap gain b_0, b_1, or b_2 and an input pulse amplitude. The input sequence has eight elements (as previously observed, four of these are zero). Label these x(1), x(2), …, x(8) (you should find it convenient to choose x(1) = x(2) = x(7) = x(8) = 0). Label them on your Graph 6-2.
13. Numeric indexing is useful as an aid in looking for the general pattern. The next step is to use symbolic indexing so that the set of formulas can be condensed into one. Here are some of the formulas as they appear with numeric indexing:

    y(3) = b0 .x(3) + b1 .x(2) + b2 .x(1)

    y(4) = b0 .x(4) + b1 .x(3) + b2 .x(2)

    y(5) = b0 .x(5) + b1 .x(4) + b2 .x(3)

    y(6) = b0 .x(6) + b1 .x(5) + b2 .x(4)

14. Write down the formula for y(2) and y(1) ? Discuss any unexpected differences.
15. The general pattern is readily apparent. With symbolic indexing, we can replace these with the single formula

    y(n) = b0 .x(n) + b1 .x(n -1) + b2 .x(n - 2)

    where n represents the position in the sequence as a discrete time index. Before, it was found that the unit pulse response h(k) for this system is h(0) = b0 , h(1) = b1 , h(2) = b2 . Hence we can express the formula for y(n) as

    y(n) = h(0) .x(n) + h(1).x(n - 1) + h(2).x(n - 2)

    i.e.,y(n) = sum over range of k {h(k).x(n - k)}

    This simple formula is known as convolution.

16. For the discrete signal case we have implemented it is also known as the convolution sum or the superposition sum. It can also be expressed in this more compact form:

    y(n) = $\sum_{k=0}^{n} h(k)x(n-k)$

    y(n) = x(n) * h(n) where * represents the “convolution” operator.

    It provides a neat way of packaging the procedures you carried out in your investigation, and it allows the range of k to be extended to any desired value. It is the mathematical representation of the interaction between the input sequence and the unit pulse response of the system.

17. Note in particular the term x(n-k). Since we are computing in terms of k, this is a time reversed version of x(k). Explain why this term is reversed and what does this mean?
18. For the continuous signal case, the summation is achieved using integration and is known as the convolution integral or the superposition integral. In brief, the convolution integral for continuous signals comes about by increasing the number of discrete samples whilst reducing

their width to a limit of 0, such that the sum of many products can be represented by the integration of the product. Its equation is y(t) = $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)dt$

19. Repeat this process with a new system response. Let each tap be equal to 0.333 ie: $b_0=b_1=b_2=0.333$.
20. What is a common label for this response?

Sinewave Input

Up to this point relatively short input sequences have been used. This made it easy to trace the passage of the pulses through the system since output segments remained distinct and could be readily referenced to the corresponding input segment. However, it needs to be considered whether the formula that obtained on this basis is also valid in the more general and usual situation when the input signal is an ongoing stream without breaks.

For this purpose we will go over the same procedure using a sinewave as input. This means there is no "natural" reference marker, hence you will need to designate one.

Nevertheless, keeping track of time points will be straightforward since the signal is periodic and the number of samples per period is relatively small.

1. We use the setup of the previous part, and re-use “full sinewave” output at DAC-0 by bypassing the RECTIFIER.
2. As before, since the sinewave frequency is a submultiple of the PULSE GENERATOR clock, no slippage occurs in the scope displays.
3. Proceed as in the previous part, connecting only one of the three adder inputs. As before, observe and record each of the separate output sequences for each input in turn (Graph 6-3).
4. As you would expect, these are scaled and delayed replicas of the input, i.e. they are sampled sinewaves. Carry out spot checks to verify that amplitude and phase relative to the input are correct. (Reminder, since there are eight samples per period, the phase shift corresponding to a unit delay is 45 degrees.)
5. For each time slot, add up the contributions of the three output records obtained above and plot the result. Verify that this agrees with the output signal produced when all the input leads are reconnected to the adder.
6. Now we are ready to revisit the formula obtained in the previous part of the experiment. Proceed as before and show that the formula remains valid for this case.
7. Confirm that the eight pulses making up one period of the output sequence represent samples of a sinewave. A straightforward method is to exploit the sum of squares identity. Since there are eight samples per period, you can match pairs of samples that are 90 degrees apart (how many pairs can you find?). Note that knowledge of the peak amplitude is not essential for this (all that is needed is to show that the sum of the squares is the same for each pair). Show your working for the sum of squares analysis?
8. Use the FUNCTION GENERATOR tuned to 100 Hz to provide the alternative unsynchronized sinewave input. Set it to an amplitude of 2Vpeak (4V pp).
   a. The resulting slippage effectively produces an interpolation of the samples -- a useful exploitation of something that is usually unwanted!

b. Now that we have a simple way of displaying the input and output sinewaves, an interesting additional item to examine is the theoretical verification of the measured output/input amplitude ratio and phase shift. The peak values are clearly apparent, so the amplitude ratio measurement is straightforward. Similarly, the now discernible zero crossings can be used for the phase shift measurement.
c. The math for the theoretical verification is done by means of the application of the formula for reducing the sum of sinusoids.

Mystery Application

Here we explore a special application of convolution. The setup is the same as for the previous part, except for new tap gains in the delay line. Two sets will be tested.

- The first is $b_0 = 0.3$, $b_1 = 0.424$, $b_2 = 0.3$
- The second set is $b_0 = -0.3$, $b_1 = 0.424$, $b_2 = -0.3$

1. When the tap gains have been adjusted to the first set of values display the output, and measure the amplitude and phase relative to the input as in the previous part. Confirm that this is in agreement with theory.
2. Alter the tap values to the second set of values and repeat the measurements. Is the outcome predicted by theory?
3. Vary the frequency of the sinewave over a suitable range to demonstrate that sinewave inputs with frequencies near 100 Hz are heavily attenuated. Why is the outcome obtained described as filtering?

DATA/OBSERVATIONS

Graph 6-1 Unit Pulse Summation

INSTRUCTOR'S INITIALS: DATE:

Graph 6-2 Inputs and Sampled Outputs

INSTRUCTOR'S INITIALS: DATE:

Graph 6-3 Inputs and Sampled Outputs

INSTRUCTOR'S INITIALS: DATE:

POSTLAB

Post-Lab questions must be answered in each experiment's laboratory report.

1. Find out the name given to systems that have the structure in Figure 6-1, i.e. without feedback. Consider an alternative discrete-time system that has a single delay element with feedback. Show how to apply the convolution formula in this case.
2. Consider the condition(s) required to avoid infinite output values in applications where the unit pulse response is not time limited. Show how to avoid an unstable output in the feedback system in the previous question.
3. Consider a process that consists of taking a running average of a data sequence, such as atmospheric pressure. Suppose we do this by taking the sum of 50% of the middle value, and 25% of the preceding and following values. Can this process be described as convolution? If so, write down the unit pulse response.
4. Using integration instead of discrete summation, write down a continuous-time version of the convolution formula in terms of the system impulse response.