

UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical and Computer Engineering

EXPERIMENT 2 – LINEAR AND NONLINEAR SYSTEMS

OBJECTIVES

After completing this experiment, the student will have a better understanding of the special role of sinusoids in linear systems. Knowledge and experience of testing for linearity.

MATERIALS/EQUIPMENT NEEDED

NI ELVIS II

EMONA SIGEx Signal & Systems add-on board

Assorted patch leads

Two BNC – 2mm leads

INTRODUCTION

Systems are used to process signals to allow modification or extraction of additional information from signals. A system may consist of physical components or of an algorithm that computes an output signal from an input signal. Roughly speaking, a system is a function mapping input signals into output signals. The study of properties of systems is important because it helps understand and interpret the behavior of certain types of systems.

In this lab, we will study linearity of some basic systems. The trademark of linear systems is that the outputs are a linear function of the inputs. Mathematically, a function is said to be linear if it possesses two properties: homogeneity and additivity. Homogeneity means that the output is proportional to the input. Additivity means that the output due to two or more inputs can be found by adding the obtained when each input is applied separately. These two properties, taken together, are often referred to as the principle of superposition. In mathematical form these properties are written as

Homogeneity: $a \cdot y(x) = S(a \cdot x)$

Additivity: $y_1(x_1) + y_2(x_2) = S(x_1 + x_2)$

One of the systems that are going to be studied in this lab is the voltage controlled oscillator (VCO). The frequency of such an oscillator can be varied by an amount proportional to the magnitude of an input (control) voltage. Such oscillators, in the form of an integrated circuit, have very linear characteristics over a frequency range which is a significant percentage of the center frequency.

PRELAB

1. Write down a formula to express the square of a sinusoid in terms of a double angle argument.
2. What is the meaning of differential linearity?
3. Consider the two conditions for linearity for a system S defined by $y = S(x)$. How would you apply these formulas in testing systems for linearity in this Lab ? How many replicas of the system are needed for the additivity test?

PROCEDURE

Setting up the NI ELVIS/SIGEx Board

1. Turn on the PC (if not on already) and wait for it to fully boot up.
2. Turn on the NI ELVIS unit but not the Prototyping Board switch yet. You should observe the USB light turn on (top right corner of ELVIS unit).
3. Turn on the NI ELVIS Prototyping Board switch to power the SIGEx board. Check that all three power LEDs are on. If not call the instructor for assistance.
4. Launch the “SIGEx Rx_x.exe” Main VI.
5. When you’re asked to select a device number, enter the number that corresponds with the NI ELVIS that you’re using.
6. You’re now ready to work with the NI ELVIS/SIGEx bundle.
7. Select the Lab 4 tab on the SIGEx SFP.

Note: To stop the SIGEx VI when you’ve finished the experiment, it’s preferable to use the STOP button on the SIGEx SFP itself rather than the LabVIEW window STOP button at the top of the window. This will allow the program to conduct an orderly shutdown and close the various DAQmx channels it has opened.

Simple Systems and Checking for Linearity

In this Part students will study some simple systems and check for linearity. First the LIMITER, next the RECTIFIER.

1. Connect the FUNC OUT terminal of the FUNCTION GENERATOR to the LIMITER input (See Figure 2-1). Connect CH0 to this input. Connect CH1 to the LIMITER output. The settings are as follows:
 - a. FUNCTION GENERATOR: FREQUENCY=1000; AMPLITUDE: 1 V pp; SINE waveform selected
 - b. LIMITER: DIP switch to OFF:OFF for “medium” clipping.
 - c. SCOPE: Timebase: 4ms; Rising edge trigger on CH0; Trigger level=0V

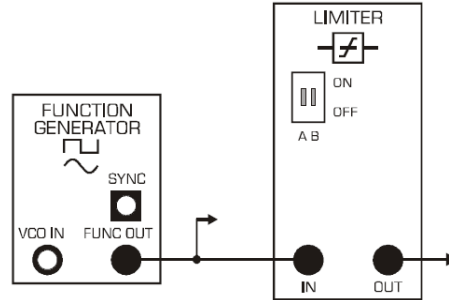


Figure 2-1 Diagram of “Clipping” System

2. Adjust the amplitude of the SINE waveform from 1 V_{pp} to around 10 V_{pp}. Take various readings throughout this range and document in the Table 2-1. **HINT:** Use the cursors on the graph of TAB 4 to quickly take peak readings, then double.
3. Connect the FUNC OUT terminal of the FUNCTION GENERATOR to the RECTIFIER input. Connect CH0 to this input. Connect CH1 to the RECTIFIER output, as per Figure 2-2. The settings required are as follows:
 - a. FUNCTION GENERATOR: FREQUENCY=1000; AMPLITUDE: 1 V_{pp}; SINE waveform selected
 - b. SCOPE: Timebase 4ms; Rising edge trigger on CH0; Trigger level=0V

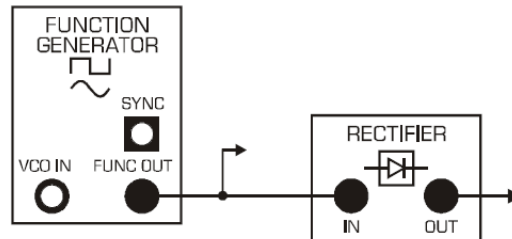


Figure 2-2 Diagram of “Rectifying” System

4. Adjust the amplitude of the SINE waveform from 1 V_{pp} to around 6 V_{pp}. Take various readings throughout this range and document in the Table 2-1. As the output signal appears only to be positive going, measure its positive amplitude (which is also its total peak to peak value). Show that this system satisfies the homogeneity test over part of the input range (i.e. where the range excludes the zero input point).
5. Connect the FUNC OUT terminal of the FUNCTION GENERATOR to the MULTIPLIER inputs. Connect both inputs in parallel. Connect CH0 to this input. Connect CH1 to the MULTIPLIER output, as shown in Figure 2-3. The settings required are as follows:
 - a. FUNCTION GENERATOR: FREQUENCY=1000; AMPLITUDE: 1 V_{pp}; SINE waveform selected
 - b. SCOPE: Timebase 4ms; Rising edge trigger on CH0; Trigger level=0V

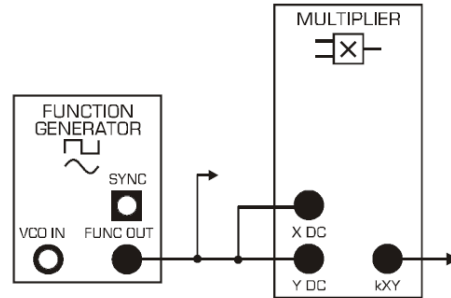


Figure 2-3 Diagram of “Multiplying” System

The Voltage Controlled Oscillator as a System

In this part students will explore the output-input characteristics of the VCO (voltage controlled oscillator) and apply linearity tests.

1. Connect the circuit shown in Figure 2-4. Settings are as follows:
 - a. FUNCTION GENERATOR: FREQUENCY=2000 Hz; AMPLITUDE: 4 V pp; SINE waveform selected
 - b. MODULATION TYPE: FM
 - c. SCOPE: Timebase 4ms; Rising edge trigger on sinewave output; Trigger level=0V

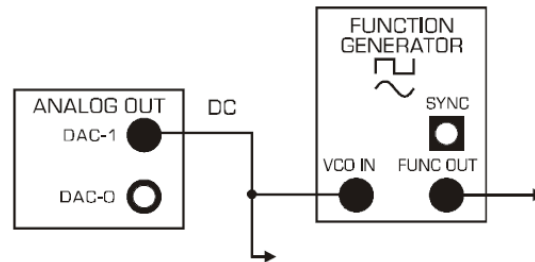


Figure 2-4 Diagram of the model for investigation of VCO as a system

2. Display both the input DC voltage and the output sinusoid. Ensure that the scope is triggering on the sinusoid signal for a stable display.
3. Observe the effect of varying the DC input voltage. Document output frequency vs input DC voltage in the Table 2-3. (The scope can be used to measure the DC voltage and frequency).
4. Consider whether the VCO is a linear system, ie in terms of output frequency vs input DC voltage. For this purpose should you focus on incremental frequency rather than absolute frequency ? (cf Differential Linearity)
5. Replace the DC signal with the signal output from the MULTIPLIER, where the input to the MULTIPLIER is both the DC voltage at DAC-1 and the fixed sinewave at DAC-0 (See Figure 2-5).

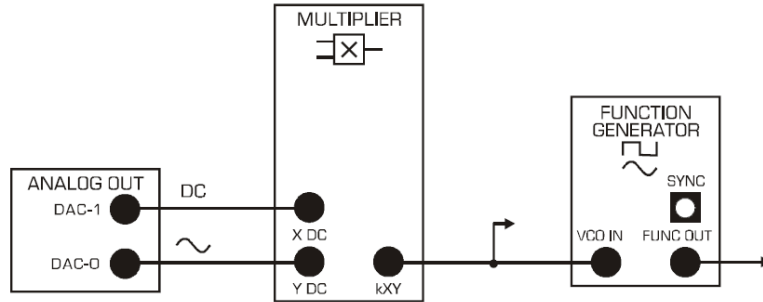


Figure 2-5 Diagram of the model for investigation of VCO as a system with rapidly “varying” input level

6. This effectively gives you amplitude control over the sinewave by using the DC voltage as the multiplication factor. Observe the effect of varying the DC input voltage on the MULTIPLIER output, as well as the VCO output.
7. Observe the VCO output. Consider possible applications (hint: synonym for ‘varying’ is ‘modulating’). View the input and output of the VCO with both scope leads. Trigger the scope on the input signal for a stable display.

Note: As a general rule, it is best to trigger the scope on the lowest frequency signal being displayed. This causes the scope to display the slower signal with stability, and have the faster signal moving relative to that. In this case, although the two signals displayed are not synchronized, it is easy to see the influence of the lower frequency signal on the VCO output. Try both ways and see if you agree.

Feedback System

In this part students will apply both linearity tests to a simple feedback system implemented with the LAPLACE module. First some basic exploration of the operation of the INTEGRATOR will be performed.

1. Connect the circuit shown in Figure 2-6. Settings are as follows:
 - a. FUNCTION GENERATOR: FREQUENCY=1000 Hz; AMPLITUDE: 4 V pp; SINE waveform selected
 - b. MODULATION TYPE: NONE
 - c. LIMITER DIP switches: DOWN:DOWN INTEGRATOR RATE DIP switches: UP:UP
 - d. SCOPE: Timebase 2ms; Rising edge trigger on CH0; Trigger level=0V

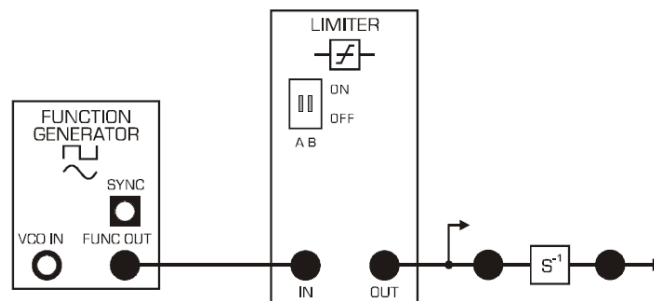


Figure 2-6 Diagram of the model for sawtooth generation

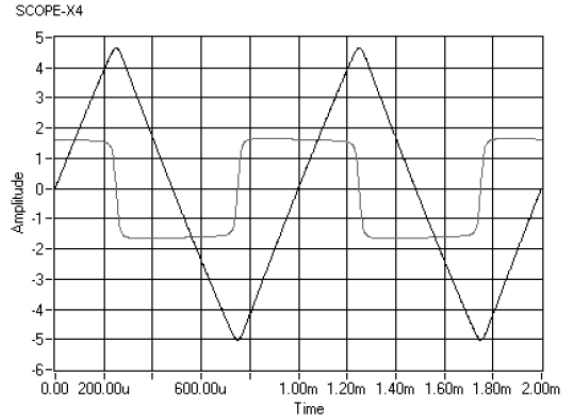


Figure 2-7 Scope Display for sawtooth generation

2. Measure the p-p amplitude of the output sawtooth waveform. Calculate the slope (slew rate) of the sawtooth and use this to write a formula for the INTEGRATOR output in terms of the input. What is the formula for the INTEGRATOR output?
3. Repeat this for the other three available INTEGRATOR rate settings. If a particular INTEGRATOR rate is too high, then note this. What are the formulas for the other INTEGRATOR rate settings?
4. Connect the modules according to Figure 2-8 and Figure 2-9. Settings are as follows:
 - a. FUNCTION GENERATOR: FREQUENCY=500 Hz; AMPLITUDE: 2 V pp; SQUARE waveform
 - b. selected
 - c. MODULATION TYPE: NONE INTEGRATOR RATE DIP switches: UP:UP
 - d. SCOPE: Timebase 4ms; Rising edge trigger on CH0; Trigger level=0V
 - e. ADDER GAINS: $b_1=1.0$; $b_2= - 1.0$
 - f. View the output and input of the INTEGRATOR as CH0 and CH1 respectively.
 - g. Observe the displayed waveforms and satisfy yourself that they represent step responses. Note that these responses are made up of exponential segments.
 - h. Examine the waveform at the ADDER output (input to the INTEGRATOR), and confirm that it looks like the impulse response corresponding to the step response at the INTEGRATOR output.

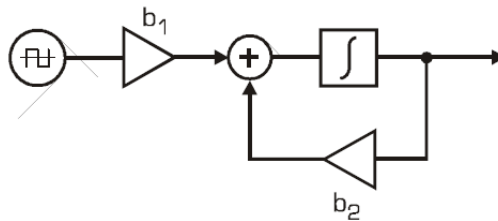


Figure 2-8 Block Diagram for a feedback System

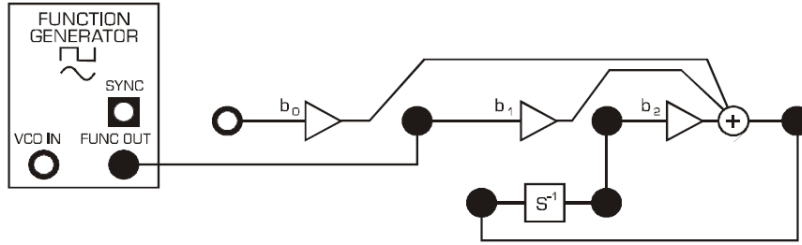


Figure 2-9 Connection Diagram for a feedback System

5. On the scope, measure the time taken for the exponential curve to decay by 50% and use this to calculate the time constant.
6. Observe the effect of varying the b2 gain, and check that this is in general agreement with the predictions of the theoretical formulas. In particular, take a close look at b2 values of 0, -0.1, -0.2, ... and confirm that you can see effect addition (or subtraction) of the output from the input, as you subtly introduce feedback.
7. Change the input amplitude and confirm that the system satisfies the homogeneity condition for linearity. Switch the FUNCTION GENERATOR back to SINEWAVE output to demonstrate that the result is the same with a sinusoidal input.

Testing for Additivity

In this section students will test for the additivity condition. The block diagram below depicts a model for this, that requires three identical copies of the system under test ie: $s(a)$, $s(b)$ and $s(c)$. Test signal x_1 is a sinewave from ANALOG OUT DAC-0; x_2 is a squarewave of opposite phase from ANALOG OUT DAC-1.

1. Select the appropriate signals as outputs from the ANALOG OUTPUT block by switching ON the “Part 4 Signal Select” button on TAB “Lab 4”
 - a. FUNCTION GENERATOR: FREQUENCY=1000 Hz; AMPLITUDE: 4 V pp; SINE waveform selected
2. As a first step, confirm that each $S(x)$ is identical, by inputting the same signal into each and comparing their outputs. You can use the square signal from DAC-1 as an interesting and familiar source. Setup the INTEGRATOR RATE DIP switches as UP:UP
3. Patch up the 2 models as shown in Figure 2-11. Settings are as follows:
 - a. INTEGRATOR RATE: UP:UP
 - b. ADDER GAINS: $a_0=a_1=b_0=b_1 = +1.0$; $a_2=b_2=0$;
 - c. SCOPE: Timebase = 4ms, Trig level = 0V

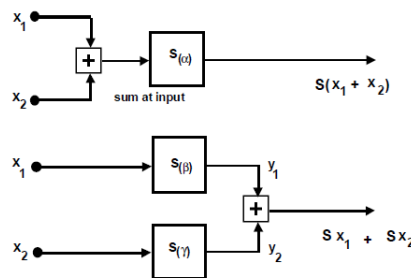


Figure 2-10 Test Method using Matched Copies of the System

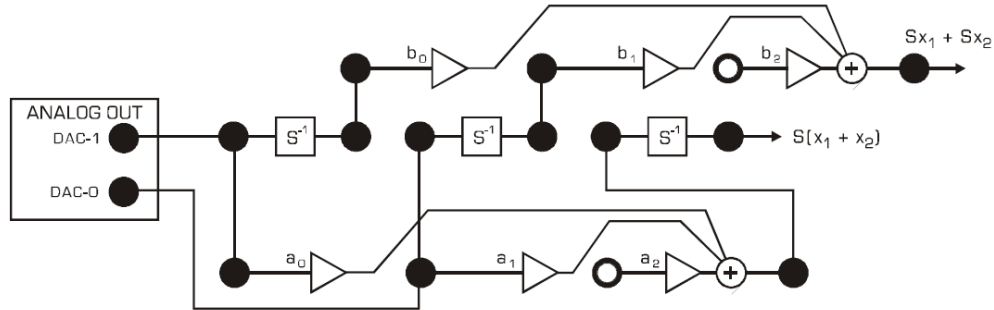


Figure 2-11 Model for Generating the $S(x_1+x_2)$ and Sx_1+Sx_2 arrangements simultaneously

4. Focus firstly on system $S(x_1 + x_2)$. View all points of the system and confirm that the signals are as expected.

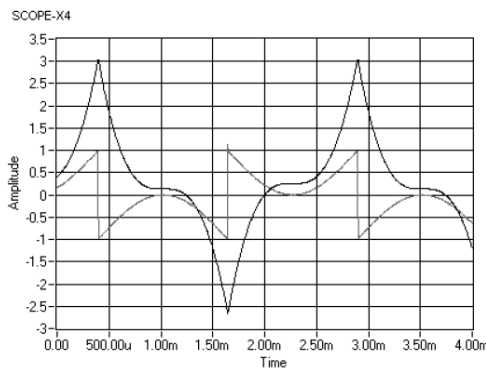


Figure 2-12 Example of (x_1+x_2) before and after processing by system S

5. Now view the signals of system $Sx_1 + Sx_2$ and confirm that all signals are as expected. Produce an accurate record each of the systems output waveform on Graph 2-1 in the data section.
6. Vary the gains of one system and observe the effect. Remember to ensure that gains $a_1=b_1$ and $a_0=b_0$ to maintain a consistent comparison.

Frequency Response

In this section students will investigate the use of sinewaves and squarewaves to measure frequency response.

1. Measure the frequency response of BBLPF with a square wave. Use the set up in Figure 2-13. Use the square wave available at the FUNCTION GENERATOR block, FUNC OUT terminal, by selecting SQUAREWAVE output (this signal is a bipolar squarewave, as opposed to the TTL level squarewave available at the SYNC output). Explain your criterion for the amplitude measurement at the output (specifically, to deal with the shape changes and ringing oscillations). Enter your readings into Table 2-4.
2. Next, try with sinewaves. The measurement is very straightforward. Unlike squarewaves, the sinewaves emerge at the output with no apparent change of shape. The issue of amplitude measurement criterion does not arise. Enter your readings into the same table.

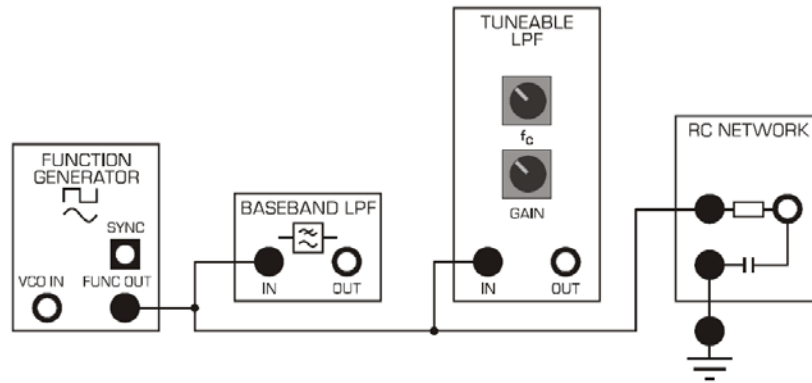


Figure 2-13 Diagram for investigating squarewave and sinewave frequency response

DATA/OBSERVATIONS

Table 2-1 Measurements of the LIMITER and RECTIFIER Systems

Input Amplitude (Vpp)	LIMITER Amplitude (Vpp)	RECTIFIER Amplitude (Vpp)

INSTRUCTOR'S INITIALS: DATE:

Table 2-2 Measurements of the MULTIPLIER System

Input Amplitude (Vpp)	Multiplier Amplitude (Vpp)

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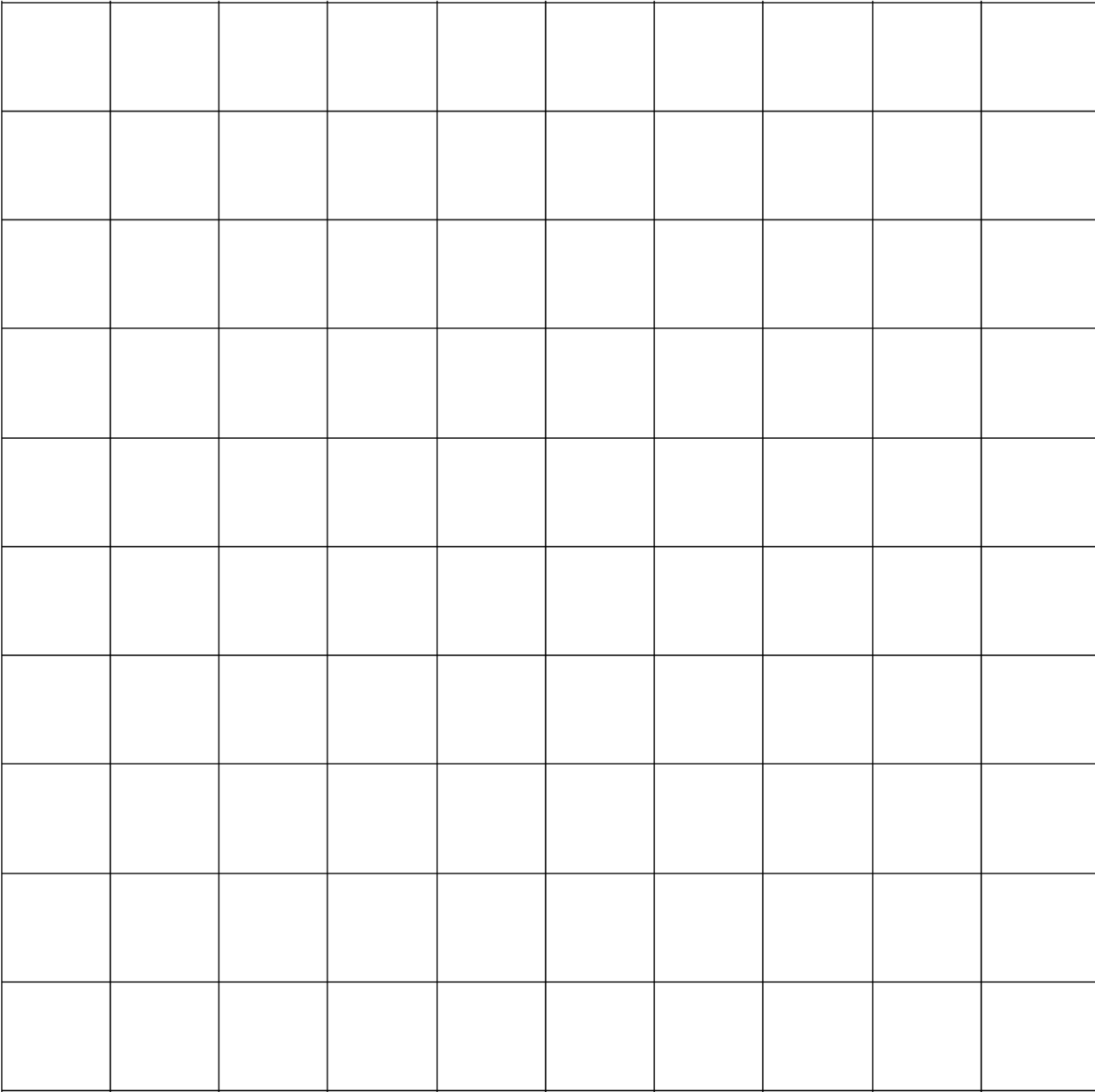
Table 2-3 Measurements of the VCO as a System

Input DC Voltage (Vpp)	VCO Output Frequency (Hz)

INSTRUCTOR'S INITIALS:

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Graph 2-1 Additivity Signals



INSTRUCTOR'S INITIALS:

DATE:

Table 2-4 Measurements of the Square Wave and Sine wave investigation

Input Frequency (Hz)	Square Wave Output (V)	Sine Wave Output (V)

INSTRUCTOR'S INITIALS:

DATE:

POSTLAB

Post-Lab questions must be answered in each experiment's laboratory report.

Simple Systems and Checking for Linearity

1. Does the CLIPPER system satisfy the scaling test for linearity? Show your reasoning.
2. Does the RECTIFIER system satisfy the scaling test for linearity? Show your reasoning. Consider the meaning and applications of the term 'differential linearity' (also known as 'incremental linearity').
3. Does the MULTIPLIER system satisfy the scaling test for linearity? Show your reasoning.

The Voltage Controlled Oscillator as a System

4. Is the VCO a linear system? Explain your reasoning.
5. What applications could the VCO with varying output frequency be used for?

Feedback System

6. What are the formulas for the INTEGRATOR output all the different rate settings?
7. Use the value of the b2 gain, and INTEGRATOR constant you measured above to determine the time constant of the exponential responses. Compare this with the value obtained from your measurement.
8. Write a differential equation for this first-order feedback system. Assume initial conditions are zero. Show that with a sinusoidal function of time as input, the output is also sinusoidal. Show that this also happens when the input is a complex exponential. Which special property of complex exponential functions provides the key?

Testing for Additivity

9. Does the outcome indicate that the linearity conditions have been met for these two test inputs?
10. Does the outcome during variation indicate that the linearity conditions are still maintained for these two test inputs?

Frequency Response

11. How were you able to use the square wave for this test?